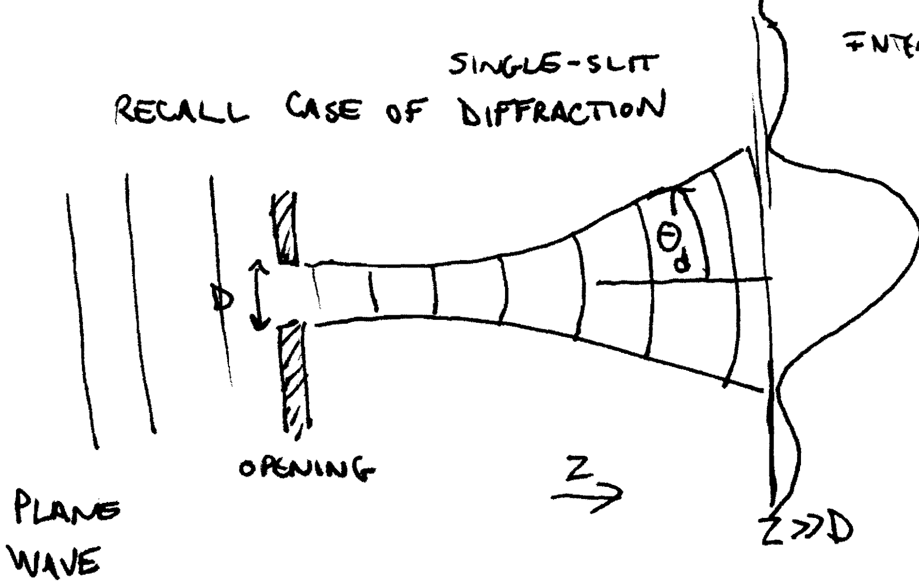
LECTURE 2

SPATIAL OR TRANSVERSE COHERENCE

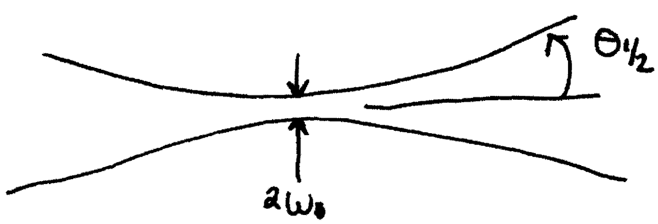
RECALL SINGLE-SLIT CASE OF DIFFRACTION



$\theta_d \sim \dfrac{\lambda}{D}$ DIVERGENCE ANGLE - MOST OF LIGHT WITHIN CONE

FOR GAUSSIAN LASER BEAM

$\omega_0 \equiv$ INITIAL BEAM RADIUS



$\theta_{1/2} = \dfrac{\lambda}{\pi \omega_0}$ - FROM GAUSSIAN BEAM PROPAGATION DERIVATION

ACTUAL LASERS CHARACTERIZED BY $\theta_c \sim \dfrac{\lambda}{D_c}$ ← TRANSVERSE COHERENCE LENGTH

INCOHERENT $\equiv$ SMALL D_c, RAPID SPREAD $D_c < D$

COHERENT $D_c = D$ - BEST YOU CAN DO - "DIFFRACTION LIMITED"

DEFINE $M^2 = \dfrac{\text{DIVERGENCE OF ACTUAL LASER}}{\text{DIVERGENCE OF IDEAL LASER}} = \dfrac{\theta_c}{\theta_d} = \dfrac{D}{D_c}$

M^2 CLOSER TO 1.0 MEANS CLOSER TO IDEAL LASER.

GAUSSIAN BEAMS

MAXWELL'S EQUATIONS CAN BE RE-ARRANGED TO YEILD THE SO-CALLED "WAVE EQUATION"

$$\nabla^2 \vec{E}(r,t) = \frac{1}{V^2}\frac{\partial^2 E}{\partial t^2} \qquad \text{where } V = \frac{c}{n} \qquad (1)$$

TRY SOLUTION OF FORM

$$E(r,t) = \underbrace{U(x,y,z)}_{\text{SOME FUNCTION WHICH DETERMINES TRANSVERSE + SPATIAL PROFILE}} \; \underbrace{e^{i(kz-\omega t)}}_{\text{PLANE WAVE}} \qquad (2)$$

SUB OUR GUESS INTO WAVE EQUATION (1)

$$e^{i(kz-\omega t)}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\frac{\partial^2 u}{\partial z^2}}_{\text{SINCE ATT. VS Z IS SMALL}} + 2ik\frac{\partial u}{\partial z} - \underbrace{\left(k^2 - \frac{\omega^2}{c^2}\right)}_{\omega \equiv kc \text{ SO TERM IS } 0} u\right] = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2ik\frac{\partial u}{\partial z} = 0 \qquad (3)$$

MAKE EDUCATED GUESS FOR U BASED ON EXPECTED TRANSVERSE BEHAVIOUR

$$\tilde{u}(x,y,z) = E_0\, e^{i(p(z) + k[x^2+y^2]/2q(z))}$$

WHERE $p(z)$ AND $q(z)$ ARE GENERAL, UNKNOWN FUNCTIONS.

PLUGGING INTO (3) AND REARRANGING

$$\left[\underbrace{\left(\frac{2ik}{q} - 2k\frac{\partial p}{\partial z}\right)}_{} + \underbrace{\left(\frac{k^2}{q^2}\frac{\partial q}{\partial z} - \frac{k^2}{q^2}\right)}_{}(x^2+y^2)\right]\tilde{u} = 0$$

CAN TREAT THESE COEFFICIENTS SEPERATELY

SO $\frac{\partial p}{\partial z} = \frac{i}{q}$ AND $\frac{\partial q}{\partial z} = 1$

MAKE A GOOD GUESS THAT $\frac{1}{\tilde{q}} = \frac{1}{R} + i\frac{\lambda}{\pi w^2}$ (4)

WHERE R IS RADIUS OF CURVATURE

W IS "SPOT SIZE" IS BEAMS HALF-WIDTH WERE IRRADIANCE HAS DROPPED TO $1/e^2$ OF ON-AXIS VALUE

PLUGGING (4) INTO (2) GIVES

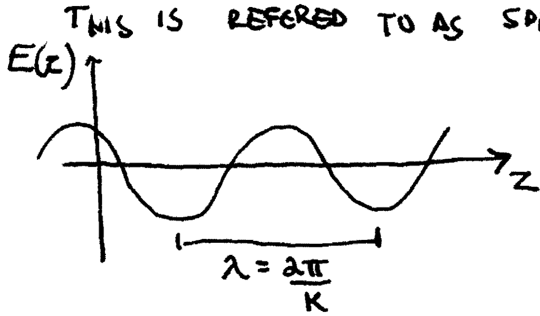
$$\tilde{U}(x,y,z) = E_0\, e^{ip(z)}\, e^{ik(x^2+y^2)/2R(z)}\, e^{-(x^2+y^2)/w^2}$$

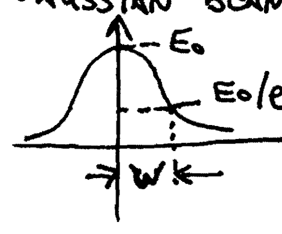
SO FROM (2)

$$E(x,y,z,t) = E_0\, \underbrace{e^{ik(x^2+y^2)/2R(z)}}_{\text{DESCRIBES THE WAVEFRONT}}\; \underbrace{e^{-(x^2+y^2)/\omega^2(z)}}_{\text{DESCRIBES TRANSVERSE PLANE}}\; \underbrace{e^{i[kz+p(z)-\omega t]}}_{\text{Z-DEPENDENT PHASE INFORMATION}}$$

P(Z) YET TO BE DETERMINED

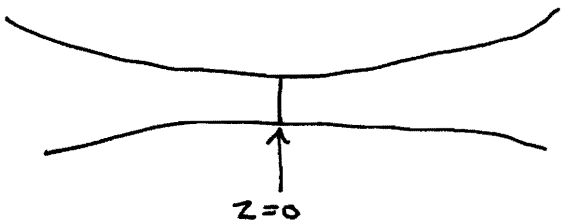
THIS IS REFERED TO AS SPHERICAL GAUSSIAN BEAM





HOW CAN WE USE THIS SOLUTION?

SUPPOSE AT SOME POINT, $Z=0$ WE HAVE A PLANAR WAVEFRONT



THEN $R \to \infty$

SO FROM (4) $\frac{1}{\tilde{q}_0} = \frac{i\lambda}{\pi W_0^2}$ or $\tilde{q}_0 = \frac{-i\pi W_0^2}{\lambda}$

NOW, SINCE $\frac{\partial q}{\partial z} = 1$ THEN $\tilde{q}(z) = \tilde{q}(0) + Z$ (SINCE $q(0) = q_0$)

SO $\tilde{q}(z) = z - i\frac{\pi_0^2}{\lambda}$

WRITING IN RECIPROCAL FORM, AND RATIONALIZING DENOMINATOR

$$\frac{1}{\tilde{q}(z)} = \frac{z}{\left(z^2 + \frac{\pi^2 W_0^4}{\lambda^2}\right)} + i\,\frac{\pi W_0^2/\lambda}{\left(z^2 + \frac{\pi^2 W_0^4}{\lambda^2}\right)}$$

COMPARE THIS TO OUR INITIAL GUESS FOR $\frac{1}{\tilde{q}(z)}$

$$\frac{1}{\tilde{q}(z)} = \frac{1}{R} + i\,\frac{\lambda}{\pi W^2}$$

IT ~~IS~~ CAN SHOWN

$$R(z) = Z\left[1 + \left(\frac{\pi W_0^2}{\lambda Z}\right)^2\right] \qquad (5)$$

$$W^2(z) = W_0^2\left[1 + \left(\frac{\lambda Z}{\pi W_0^2}\right)^2\right] \qquad (6)$$

WHERE $R(z)$ AND $W(z)$ ARE ACTUAL PHYSICAL PARAMETERS

NOTE: IF $R(z) = Z$ ONLY, THEN WE WOULD HAVE SPHERICAL WAVEFRONTS CENTRED AT $Z=0$.
HOWEVER IF $Z \gg \frac{\pi W_0^2}{\lambda}$ THEN $R(z) = Z$ AND WE DO HAVE. THIS IS CALLED "FAR FIELD"

AND ALSO $W(z) = \frac{\lambda Z}{\pi W_0}$ (LINEAR WITH Z), $\theta_{FF} \doteq \frac{\lambda}{\pi W_0}$